1/f noise formed by time intervals between particle detections

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LETTER TO THE EDITOR

$1/f$ noise formed by time intervals between particle detections

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Abstract. The series formed by time intervals between particle detections are generated by using the model wherein increasing and decreasing probabilities of particle number in a very short time interval are equivalent. These have $1/f$ spectra in a similar way to the series generated in our previous work by using the branching process model. The self-correlation function between the detections, however, behaves in a different way from that expected from a series with a pure $1/f$ spectrum. Owing to the anomalously large power spectral density at $f=1$, the correlation function behaves as $\mu^{-\alpha}$, where μ is a count difference between two detections.

Since the first observation of the $1/f$ spectrum in shot noise [1], $1/f$ behaviour, the frequency in time or space, has been found in many phenomena, such as changes in the weather [2], music [3], the galaxy distribution in the universe [4] etc. In order to elucidate these phenomena many theoretical works have been published [5–10], but many problems are yet opened to be solved. Recently the branching process model (BMP) [11] was applied to generate series with a $1/f$ spectrum which was formed by time intervals between successive detections of a particle existing in the system [12]. Here the frequency is related to detection counts and not to time. In that case, a system was considered where a particle is subjected to capture and binary branching reactions, and a particle detector of absorption type is placed in this system. The probabilities that n particles are found in the system at time $t>0$ and that no detection is recorded during the time interval $(0, t)$, assuming k particles exist in the system at $t=0$, were described in the closed forms. Whether a particle detection has occurred or not in a very short time interval was then decided successively using the Monte Carlo method with the above probabilities in the case that recording more than two counts in this time interval is negligible. Two series were obtained: one formed by particle numbers at successive times and the other formed by time intervals between successive detections. The spectrum of the latter series has a $1/f$ behaviour, while the former series behaves as $1/f^2$, because the correlation between count intervals is not as strong as that between particle numbers. It was, however, possible to obtain a $1/f$ spectrum only when the system is in a critical state where the absorption probability of a particle in a short time interval is equivalent to the production probability of a particle by a branching process in this interval. The above results do not answer the question of whether a particular process such as branching plays a determinant role for the $1/f$ behaviour. In the present work, therefore, we tried to obtain a $1/f$ spectrum without assuming a particular process.

A critical state can also be realised when the probability that the particle number

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N_t at a time t increases in a very short time interval Δt after t is equivalent to the probability that the number N_t decreases in Δt. This consideration (critical state model, CSM) was applied to generate the series of time intervals between successive detections. In the simulations, the mean number of detected particles N_D in Δt was assumed to be a product $N_t \times$(the detection efficiency ε of a particle in Δt). When ε is small enough, the resulting very small N_D is considered as the particle-detection probability in Δt and whether a particle detection has occurred or not in Δt can be decided successively by using the Monte Carlo method to form a series of the time intervals between successive detections. The particle number N_t was assumed to change in Δt after the time t mostly with a Gaussian distribution around N_t.

Figure 1 shows the power spectral density (PSD) of the series generated, starting from the initial particle number N_0 of 5000, for various standard deviations σ of the Gaussian distribution and detection efficiencies ε. The PSD converges to a finite value

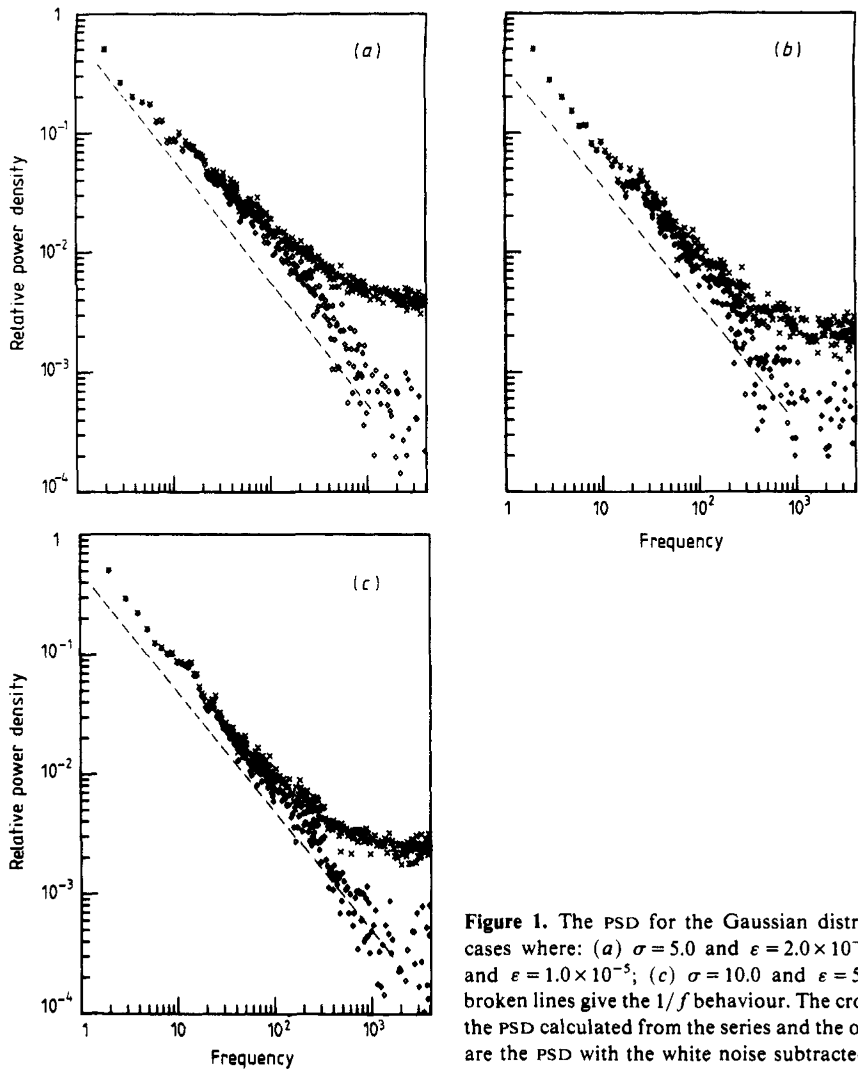


Figure 1. The PSD for the Gaussian distribution in the cases where: (a) $\sigma = 5.0$ and $\varepsilon = 2.0 \times 10^{-6}$; ($b$) $\sigma = 5.0$ and $\varepsilon = 1.0 \times 10^{-5}$; ($c$) $\sigma = 10.0$ and $\varepsilon = 5.0 \times 10^{-6}$. The broken lines give the $1/f$ behaviour. The crosses represent the PSD calculated from the series and the open diamonds are the PSD with the white noise subtracted.

(white noise component of a spectrum) in the high-frequency range, and therefore the PSD with the white noise component subtracted is also shown in figure 1. The PSD behaves evidently as $1/f$ over three decades of frequency in the figure. Shown in figure 2 are the results in the case where the particle number changes in Δt with a Lorentzian distribution, where the PSD also behaves as $1/f$. This behaviour of the PSD shown in figures 1 and 2 is similar to that in the case of simulation by the BPM [12]. Although no particular type of process occurring in the system is assumed, a particle detection has a stochastic correlation with a particle number in the system, which may cause the $1/f$ behaviour. Considering all the results obtained in the previous [12] and present works, it is possible to conclude that the $1/f$ behaviour of the PSD appears only when the system is in a critical state but the type of processes by which a critical state is realised is not important.

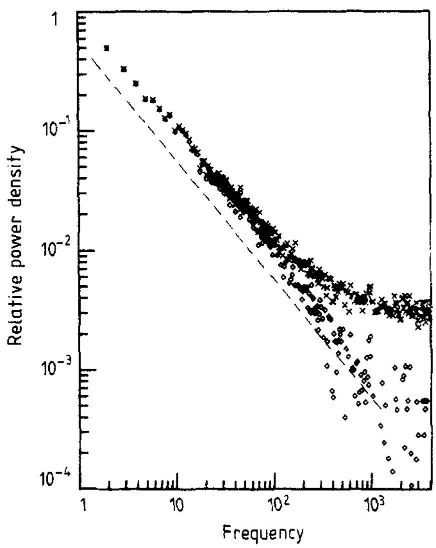


Figure 2. The PSD for the Lorentzian distribution in the case where width = 5.0 and $\varepsilon = 5.0 \times 10^{-6}$. The broken line gives the $1/f$ behaviour. The data points are defined as in figure 1.

Next the self-correlation functions $C(\mu)$ of the series generated by using the CSM and BPM are compared with each other, where μ describes a count difference between two detections. The correlation functions $C(\mu)$ obtained from figures 1 and 2 using the Wiener–Khintchine relation are shown in figure 3 in double logarithmic scales. The function $C(\mu)$ behaves as $\mu^{-\alpha}$ ($\alpha \approx 0.1$) at small μ, but the correlation decreases at large μ more slowly than expected from the $\mu^{-\alpha}$ behaviour. In figure 4 are shown the functions $C(\mu)$ calculated by using the BPM, where λ_m and Δt are the branching rate of a particle and a time step in the simulation, respectively. Owing to the limited computing time and memory, it was rather difficult to make the starting particle number N_0 as large as in the case of the CSM. On the other hand, a small N_0 is inappropriate in the CSM case because of the symmetrical distribution of the change in particle number. If the CSM simulation is started from a small N_0, a negative and unreasonable particle number will be generated frequently. The $\mu^{-\alpha}$ behaviour is evident over three decades of μ in the BPM cases, but the exponents α, being approximately 0.2, are

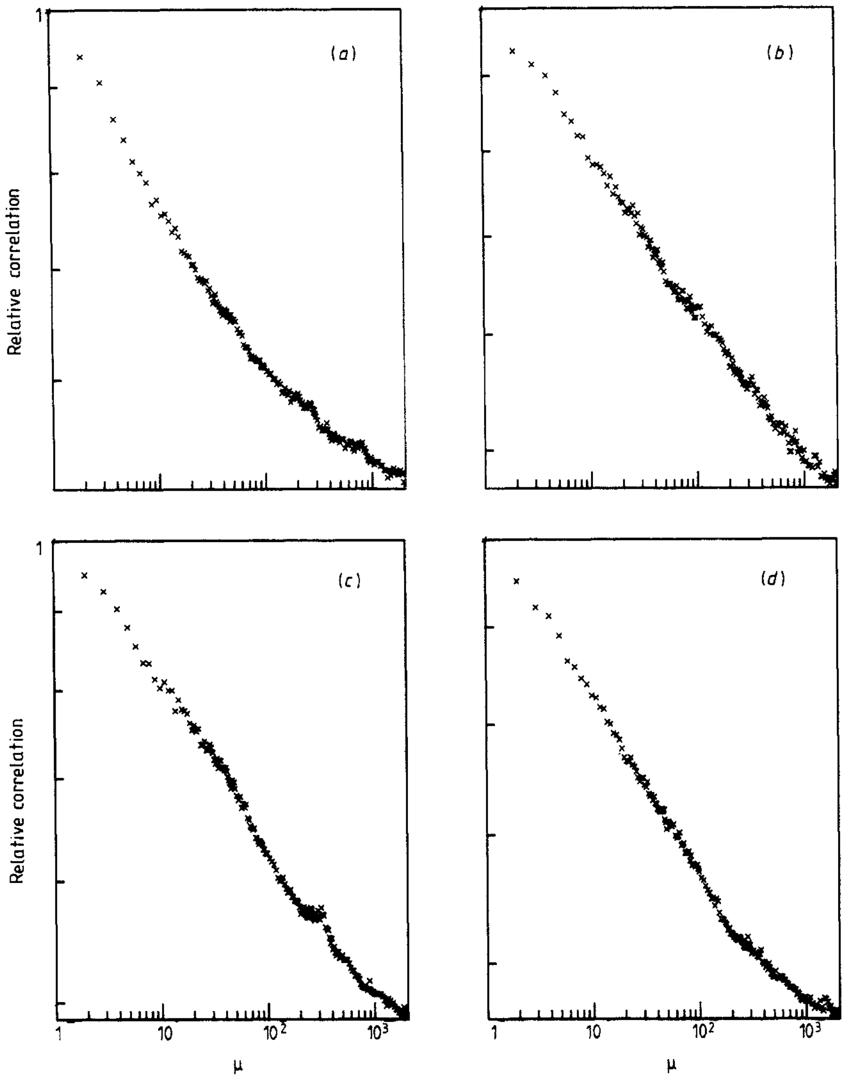
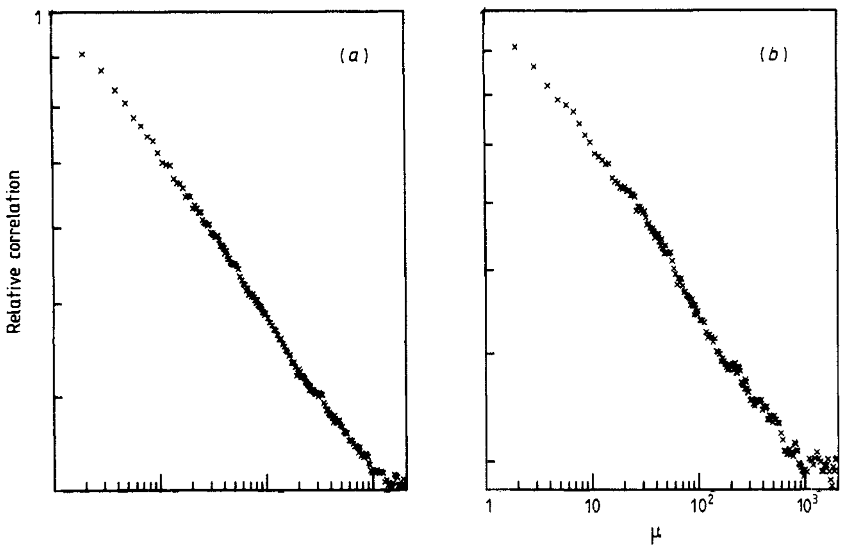


Figure 3. The correlation functions calculated from the PSD. (*a*)–(*c*) correspond to those in figure 1, and (*d*) is the function calculated from the PSD in figure 2. The exponents α are (*a*) 0.13, (*b*) 0.11, (*c*) 0.11 and (*d*) 0.09.

different from those in the CSM cases. The behaviour $\mu^{-\alpha}$ of $C(\mu)$ is probably the result of the fractal properties of the series.

The function $C(\mu)$ calculated from a pure $1/f$ distribution behaves differently from that shown in figures 3 and 4 (see figure 5). The PSD at $f=1$ for the series generated by using the CSM and BPM is several times greater than expected from the $1/f$ behaviour, although it is not shown in figure 1. In order to check the effect of the anomalous PSD at $f=1$ on $C(\mu)$, the correlation functions for these series were calculated by modifying the PSD at $f=1$ to be twice that of the PSD at $f=2$, two examples of which are given in figure 6. The similarities between the curves in figures



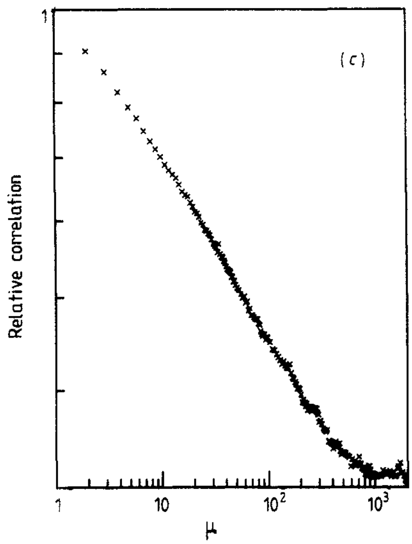


Figure 4. The correlation functions obtained by using the BPM in the cases where: (*a*) $\lambda_m \Delta t = 1.5 \times 10^{-3}$ and $N_0 = 10$; (*b*) $\lambda_m \Delta t = 2.0 \times 10^{-4}$ and $N_0 = 10$; (*c*) $\lambda_m \Delta t = 3.0 \times 10^{-3}$ and $N_0 = 1$. The estimated exponents α are (*a*) 0.17, (*b*) 0.21 and (*c*) 0.18.

5 and 6 suggest that the $\mu^{-\alpha}$ behaviour of $C(\mu)$, not expected from the $1/f$ spectrum, comes from the anomalous PSD at $f = 1$ of the series.

In table 1 is shown the ratio $R = S_1/S_f$, where S_1 and S_f are the PSD at $f = 1$ calculated from the generated series and that expected from the $1/f$ spectrum, respectively. As can be seen in table 1, the values of R in the BPM cases are approximately 3, while those in the CSM cases are larger and, moreover, are scattered. Too large PSD at $f = 1$ in the CSM cases may result in deviation from the $\mu^{-\alpha}$ behaviour.

In conclusion, a series with the $1/f$ spectrum can be generated by using the CSM as well as by using the BPM, which suggests whether a system in a critical state plays a determinant role for the $1/f$ behaviour. The correlation functions of the series behave as $\mu^{-\alpha}$, probably due to the fractal properties of the series. This behaviour of the correlation function, not expected from a series with a pure $1/f$ spectrum, is suggested

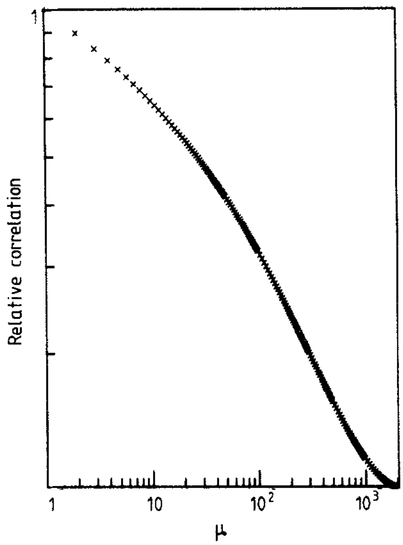


Figure 5. The correlation function calculated from a pure $1/f$ distribution.

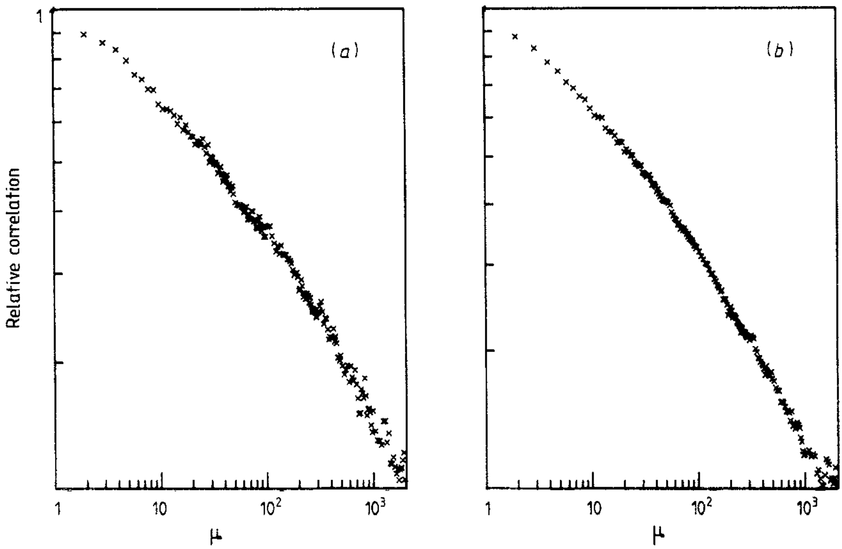


Figure 6. The correlation functions calculated by modifying the PSD at $f=1$ to be twice that of the PSD at $f=2$. (a) and (b) correspond to (b) in figure 3 and (a) in figure 4, respectively.

Table 1. The ratio $R = S_1/S_f$. (a)–(d) in the CSM series correspond to those in figure 3 and (a)–(c) in the BPM series are those in figure 4.

CSM series	R	BPM series	R
(a)	7.80	(a)	2.99
(b)	4.63	(b)	2.91
(c)	5.11	(c)	2.81
(d)	8.13		

to come from anomalously long correlation between the particle detections as reflected in the anomalously large PSD at $f = 1$.

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